Chapter 5 – Experimental Investigations of Lubricated Bearing Stiffness

# Preface

From the numerical work performed in Chapters 4 and 5, the bearing stiffness is shown to increase as a function of speed. This section outlines modifications made to the high-speed bearing test rig in Chapter 3 to enable quasi-static stiffness measurements of the rig to be performed, and qualitatively validate the findings from numerical models. To achieve this, a parameter identification methodology has been established for frequency-domain identification of bearing stiffness under high-speed operating conditions.

# List of Figures

[Figure 1 - Stiffness and Damping Coefficients in Roller Bearing Assembly [3] 6](#_Toc103875115)

[Figure 2 - Decaying vibration of linear visco-elastic system 11](#_Toc103875116)

[Figure 3 - Time-domain runout overview [1] 12](#_Toc103875117)

[Figure 4 - Modal Impact Hammer Methodology Workflow [1] 13](#_Toc103875118)

[Figure 5 - Experimental Test Rig (Confidential) 14](#_Toc103875119)

[Figure 6 - Axial Preload Device (Confidential) 16](#_Toc103875120)

[Figure 7 - Displacement Sensor Collar 17](#_Toc103875121)

[Figure 8 - Static Testing Methodology 18](#_Toc103875122)

[Figure 9 - FFT of free vibration in the vertical direction 19](#_Toc103875123)

[Figure 10 - Free vibration response in the vertical direction 19](#_Toc103875124)

[Figure 11 – Signal Runout Subtraction Results - 6000 rpm 22](#_Toc103875125)

# List of Tables

[Table 1 - Experimental Rig Components 15](#_Toc103875212)

[Table 4 - Logarithmic decrement values 20](#_Toc103875213)

[Table 3 - Oil Properties 20](#_Toc103875214)

# Introduction

Experimentally obtaining bearing stiffness and damping characteristics has been the topic of much literature surrounding dynamic bearing behaviour. This is due, in part, to insufficient accuracy of numerical predictions. It also serves a purpose of providing in-situ measurement of operational equipment used in mechanical and aeronautical engineering.

A multitude of bearing and seal types have been characterized using parameter identification. These include fluid-film journal bearings, annular seals, magnetic bearings and rolling element bearings. The experimental characterization methodology to obtain stiffness and damping properties of these components is similar. The methods are based on applying a defined excitation force, and measuring system response (displacement, velocity, acceleration). Unknown parameters of the system, mass, stiffness and damping coefficients, are then calculated by way of input-output relationships.

For this study, it is of interest to qualitatively assess the influence of the lubricant film on the component level stiffness of the bearing. As identified in the literature, the influence of lubrication on bearing stiffness above rotational speeds of 4000 rpm is to be investigated; extending the limits of investigations performed by Dietl [1] and building on his findings.

# Methodology

This section outlines the methodology used to characterize bearing stiffness using a high-speed experimental test rig.

## Types of System Identification

There exist two main methods of system identification: parametric and non-parametric. Non-parametric identification, such as Fourier analysis, do not require physical or modal system parameters to be identified. This method is typically used for fault-finding or in-situ measurement in industrial applications.

Parametric identification requires a mathematical model to be established that represents the system under investigation. System response to forced excitation is measured and compared to the response calculated using the mathematical model. Model parameters are then adjusted until the deviation between system and model is minimised. The mathematical model can take the form of a finite-element model [1], continuous models, or rigid multi-body systems. Multi-body systems are advantageous since the computational power required to establish parameter comparisons is much lower than the FE method. This is due to fewer degrees of freedom representing the physical system.

Since the aim of these investigations is to qualitatively establish bearing stiffness behaviour with speed and lubricant entrainment, this method was considered sufficient over developing a finite element model of the test system. Hence, the differential equation of motion describing a linearised system of degrees of freedom is:

Where , , and are the mass, damping and stiffness matrices respectively. The displacement vector is , and is the excitation vector.

## Time-domain parameter identification

The test rig can be represented as a two degree of freedom system undergoing lateral motion by a simplified mathematical model. Excitation force and resultant motion are measured, allowing the calculation of test impedances which represent the stiffness and damping within the system.

The system of equations best representing the two degree of freedom system are:

or

where and is the static equilibrium force required to support the rotor weight.

The unknown force coefficients to solve consist of four stiffness, and four damping, , coefficients. Four inertia force coefficients, , are included for hydrodynamic bearings and liquid annular seals working with process fluids, such as those used in industrial applications. These coefficients are solved through curve fitting the measured force-response to that predicted by the system of equations.

The force coefficients represent a linear or linearized physical system. They must therefore be determined in a test system which undergoes low amplitude motion about its equilibrium position. The time-domain method also assumes that the coefficients are independent of frequency**.** Damping is shown to vary with excitation frequency [2], which requires a method of extracting results in the frequency domain from measurements. It must be noted that these parameters are not actually measured, but estimations derived from relating the motion response to applied force to representative mechanical models.

## Frequency-domain parameter identification

More recent parameter identification techniques use frequency domain procedures [3]. This method identifies unknown parameters of a mathematical model that represents the system using experimentally obtained frequency response functions (FRFs). Arbitrary parameter values are selected for use within the mathematical model, resulting in a difference between the mathematical and experimentally obtained FRFs. The model parameters are then adjusted until the weighted deviation between mathematical and experimental FRFs reaches a minimum value.

Various minimisation methods exist, however linear regression is frequently used. These increase in complexity to deal with system noise, and include the generalised least squares method, the maximum likelihood method, and the instrumental-variable (IVF) method [4].

## Selecting the appropriate method

The test rig represents an elasto-mechanical multi-body system. Consensus from literature suggests a preference toward the frequency-domain identification techniques, especially considering the linear nature of the test setup and representative mathematical model. No mass parameters require identification since the values of these are well established from the development of the rig. This allowed for a more stable identification method.

## Methodology in this work

The frequency domain identification techniques used in this work are based on Goodwin [5] and Nordmann [6]. The dynamic force coefficients highlighted in the time-domain measurement method are estimated from transfer functions of experimentally measured displacement in response to external load. The external load is of a time-varying structure, either impact, periodic-single frequency, sine-swept or random.

Diagram

Description automatically generated

Figure 1 - Stiffness and Damping Coefficients in Roller Bearing Assembly [3]

The bearing is modelled as a point mass undergoing forced vibrations due to external excitation. Equations of motion for small amplitude excitation about an equilibrium position for a linear mechanical system are:

where are external excitation forces, is the mass, and are the bearing dynamic stiffness and damping force coefficients.Structural support stiffness and resonant damping coefficients, , are obtained from excitation of the system under dry conditions. Inertia forces and additional mass coefficients are not included for bearings.

Two independent excitation forces, and , are applied via a modal impact hammer, and the displacement of the shaft is measured. These are applied in perpendicular axes to excite the shaft laterally. The methodology of measuring impedances and hence bearing stiffness and damping is as follows:

1. Apply first test force and measure ; Apply second test force and measure
2. Obtain the discrete Fourier transform (DFT) of applied forces and displacements

note and

1. Obtain the motion ODEs in the frequency domain
2. Convert this to matrix form to separate impedances
3. Define these complex impedences (dynamic complex stiffness), as

where , for ; zero otherwise

Both real and imaginary parts of these complex impedances are functions of the excitation frequency, . The real part represents the dynamic stiffness, and the imaginary part, or quadratic stiffness, is proportional to the viscous damping coefficient. These impedances are representative of the mathematical model

1. Writing the equations of motion in terms of force, motion and impedances then becomes

for the first test; for the second test.

1. Combine the equations and rearrange

This equation represents four independent equations to solve for the four unknowns, (). The equations is valid for each excitation frequency, (). Solving for the unknowns is done via matrix multiplication

or

where

It is important to note that the test forces must be linearly independent. The forces in the second test cannot be a multiple of the first, since this would create singular matrices of forces and resulting displacements In these studies, the preferred and simplest way of ensuring this is and . This ensures reliable and repeatable results.

1. The system parameters, are determined by curve fitting. The discrete set of recorded impedances at each frequency, , are fit to the physical representative analytical functions:

The analytical function representing the system must be selected carefully. A correlation coefficient () is obtained from the curve fitting to assess the quality of the fit. For values of , it can be concluded that the analytical model chosen to represent the experimental system does not reproduce measurements. A high correlation coefficient of demonstrates that the analytical model accurately describes the test system response.

In summary, the method is to choose an analytical function that bests represents the experimental system, obtain force-response data and curve-fit the impedances and aim for high value.

System transfer functions (output/input) are often used to obtain more precise estimates of seal or bearing force coefficients [6] [7]. This process leads to curve fits of non-linear functions.

Transfer functions (displacement/force) known as test system flexibilities are derived as functions of the impedances, from the fundamental i.e.

where

Dietl [1] used averaged quantities when estimating the frequency response functions, , due to imperfect measurements.

where (\*) indicates complex conjugate quantities. denotes the real-valued auto power spectrum, while is the complex-valued cross power spectrum.

Where represents the average quantities. The power spectra are therefore averaged before computing the FRFs. The coherence function, , is used to indicate the quality of the FRF-estimation:

A value of indicates an undisturbed signal. Signals corrupted by noise will have a value of .

### Logarithmic Decrement

The damping ratio can be obtained from time-domain peak to peak analysis of the decaying impact response. An FFT is first performed to establish frequencies of interest in the system. A cut-off frequency is established, and any noise or signal outside the region of interest is attenuated.

The logarithmic decrement, , is calculated by taking the natural logarithm of the ratio of successive peak amplitudes, , cycles apart.

Diagram

Description automatically generated

Figure 2 - Decaying vibration of linear visco-elastic system

To calculate the logarithmic decrement more accurately, this can be calculated for every successive peak and averaged. Repeating this using different peak separations () allows an average value for the system to be obtained. The following relationships allow the damping ratio, *ζ*, damping, *c*, and natural frequency, *ωn*, to be obtained.

### Time-Domain Runout Compensation

To study the effects of lubrication on bearings under realistic operating conditions, excitation must be applied to a rotating shaft.

The displacement signal from the shaft has inherent waviness, due in a large part to the machining process. Eccentricity of the bearing seats and variations in rolling element and raceway profile also contribute to this, albeit to a lesser extent. These factors combine to generate mechanical runout of the shaft. The runout was first tested with a mechanical dial gauge and confirmed using the capacitive displacement sensors.

Diagram

Description automatically generated

Figure 3 - Time-domain runout overview [1]

To remove this periodic runout from the signal, a series of post-processing steps are used.

An FFT is performed to identify frequencies of interest. A Butterworth low-pass filter with cut-off set to Hz isolates the first order frequency associated with the shaft runout. The time-period in which the impact is applied is identified from the time-domain signal, and a 1 s window starting 0.2 s prior to the impact signal.

The signal is split at a point where impact response has decayed, and the signal has reached steady state. This creates two signals, one containing the hammer impact including the shaft runout, and the other is purely the shaft runout. To perform the vector subtraction of these two signals, they must be overlayed so that their periods match. This is achieved by identifying signal peaks that correspond with the maximum amplitude of the signal runout. The steady state signal is then superimposed onto the hammer signal, and vector subtraction is performed.

Due to the sampling rate and slight dynamic variation in each shaft rotation, the identified peaks are not always in the same geometric angular position when superimposed. This leads to a poor subtraction, with some effects of the runout signal observed due to the out of phase nature of the vector subtraction. To resolve this, multiple peaks are taken from the steady state signal to be used as the start point for the signal overlay. The developed processing program runs through each point until the lowest standard deviation in the subtracted signal is achieved. This results in a much clearer FRF from the signal which are absent of mechanical runout and ready to perform parameter identification.

Diagram

Description automatically generated

Figure 4 - Modal Impact Hammer Methodology Workflow [1]

# Experimental Test Rig

## Rig Overview and Modifications

The experimental rig used for initial boundary condition studies [8] was adapted for these investigations. Modifications to the preload mechanism, sensors, and shaft were required to characterize the dynamic and tribological effects of lubricants within the bearings. The rig has been developed such that all components and interfaces are of very high stiffness, with the major source of compliance within the system being the bearings. Deep groove ball bearings, 6205/C2, with an inner and outer diameter of 25 mm and 52 mm respectively sit in test brackets and support the shaft. The main components of the rig are shown in Figure 5.

|  |
| --- |
|  |
|  |

Figure 5 - Experimental Test Rig (Confidential)

Table 1 - Experimental Rig Components

|  |  |
| --- | --- |
| 1 | Electric Motor |
| 2 | Coupling |
| 3 | Torque Transducer |
| 4 | Capacitive Displacement Sensor Mount |
| 5 | Axial Preload Device |
| 6 | Bearing Test Bracket |
| 7 | Bearing |

### Shaft

To reduce deflection under radial loading, a 35 mm shaft was manufactured from 16MnCr5 steel. This was selected due to its high tensile and yield strength, high wear resistance and the ability to surface harden in accordance with BS EN ISO 683-3 standard. Bearing seats were manufactured using press-fit collars, allowing different radial preloads to be applied to the bearings depending on which bearings were under tests. For deep groove ball bearings (DGBB), these were designed as a transition fit. The shaft is polished in the region of the capacitive displacement sensors to remove profile due to surface roughness.

### Axial Load (DGBB)

Axial preload is applied to the bearing as a fixed preload against the outer bearing race. The inner races of both bearings are constrained axially against in shoulder of the shaft, with the outer race being able to displace laterally relative to their fixed position. A bearing cap constrains the motion of the shaft at the motor end of the assembly, exerting a reaction force and hence an equal relative displacement of the races. The preload is applied using either a hydraulic ram, or a screw jack with a load cell in-situ to measure precise preload. The piston and load cell design also allows for force data to be acquired simultaneously throughout rig operation, permitting investigations of the influence of the lubricant film on the axial preload.

Diagram, engineering drawing

Description automatically generated

Figure 6 - Axial Preload Device (Confidential)

### Torque Transducer

A RWT421-EC-KG torque transducer is mounted between the motor and shaft. This has a rotational speed rating of 15 000 rpm, and a torque limit of 21 Nm with resolution of 0.02 % full-scale deflection. This resolution allows for contact level variations in bearing friction to be measured, as well as torque fluctuations from the electric motor. Analog signals for torque and speed are output to the data acquisition chassis and acquired simultaneously with the radial displacement data. The motor and shaft are connected to the transducer using ROTEX GS couplings with a 98 Sh-A flexible polyurethane jaw. These are balanced to 24 000 rpm for high-speed analysis.

### Modal Impact Hammer

Impact excitation force was applied to the shaft using a Brüel and Kjær Type 8206 Impact Hammer. This has a sensitivity of 22.8 mV/N, with a full-scale measuring range of 220 N. This supplies sufficient excitation to displace the shaft at lower speeds with little dynamic effects. For higher levels of dynamic excitation and through periods of resonance, an impact hammer with a greater measuring range and therefore transfer of energy into the system is required to improve the signal to noise ratio. Data acquisition is performed in B&K Pulse Labshop, where the time and frequency domain signals are exported for use in post-processing.

### Capacitive Displacement Sensors

Lateral displacement of the shaft in two degrees of freedom are measured using Micro-Epsilon CSH1-CAm1,4 capacitive displacement sensors. These sensors provide a non-contact displacement measurement over a 1 mm range to a resolution of ±2 nm; within anticipated shaft deflections calculated from initial experiments and numerical work. Furthermore, due to the conductivity of the shaft material, variations in the material electrical and magnetic properties does not influence measurements, unlike inductive methods such as eddy current sensors.

These sensors are mounted within a collar fitted to the bearing bore in the housing. Displacement measurements obtained are therefore relative between the shaft and housing, showing purely the mechanical runout superimposed on the deviation of the shaft from its nominal geometric centre. Shims are used to offset the sensors by 0.3 mm from the shaft surface. This offset is then removed in signal processing.

Diagram

Description automatically generated

Figure 7 - Displacement Sensor Collar

### Data Acquisition and Operation

The rig is operated, and data is acquired simultaneously via a National Instruments NIcDAQ. Displacement sensor and torque transducer data acquisition is performed using MATLAB. The voltage output signal controlling the operating speed of the rig is generated using MATLAB and applied using a voltage output DAQ card. The sampling rate of the displacement measurement is 3906.25 Hz. The analog voltage signal is filtered using a low-pass anti-aliasing filter prior to digital conversion (full workflow in Figure 4).

# Results

Initial static testing was performed to develop processing methodology and obtain benchmark stiffness and damping measurements for the test rig. These tests were then expanded to dynamic (rotating) impact tests up to 8 000 rpm.

## Static Testing

The methodology used for static testing of the dry system is shown in Figure 8.

Diagram

Description automatically generated

Figure 8 - Static Testing Methodology

Impulse excitation was applied to the static shaft using the modal hammer. Digital conversion of the hammer and displacement sensor signal was performed. Offset compensation was applied to the signal, by subtracting the signal from its mean to remove the sensor offset and obtain a geometric centre. An FFT was performed to identify the impact response and determine a cut-off frequency for a low-pass filter (see Figure 9). A Butterworth low-pass filter was then applied to isolate this signal of interest. The peak-to-peak decay of the hammer impact can be observed in Figure 10.

Chart

Description automatically generated with medium confidence

Figure 9 - FFT of free vibration in the vertical direction

Chart

Description automatically generated

Figure 10 - Free vibration response in the vertical direction

The logarithmic decrement was then calculated for each successive peak with different peak separations. These measurements started at the sixth peak to allow transient effects to reduce. The damping ratio (*ζ*) for each *δ* was calculated using the following relationship and an average of those values was taken.

Table 4 - Logarithmic decrement values

|  |  |
| --- | --- |
| **N** | **ζ** |
| **6** | 0.207 |
| **7** | 0.201 |
| **8** | 0.200 |
| **9** | 0.210 |
| **…** | **…** |
| **24** | 0.185 |
| **Average** | 0.195 |

The damped natural frequency can then be found using the decay by first calculating the oscillation period, . The following relationships allow the damping ratio, *ζ*, and natural frequency, *ωn*, damping, *c,* and stiffness, , to be obtained.

## Dynamic Testing

Rotational tests were conducted using Nexbase 3080 base mineral oil to observe how the stiffness of the system is affected by the lubricant at different speed. Properties of this lubricant can be found in Table 1.

Table 3 - Oil Properties

|  |  |  |
| --- | --- | --- |
| **Lubricant** | **VG Grade** | **Viscosity @ 40°C (mm2/s)** |
| Nexbase 3080 | VG46 | 49 |

The bearings were degreased using petroleum ether in and ultrasonic bath for 5 minutes at 21˚C. This removed any existing lubricant and allowed the bearings to be inspected for wear or damage prior to testing.

A speed range of 1000 – 8 000 rpm was selected for impact testing due to the shaft stability up to these speeds. A resonance at 9 000 rpm caused high shaft displacements of similar amplitude to that of the impact hammer response. The signal-to-noise ratio is not sufficient at these speeds for accurate parameter identification and requires a modal hammer with a greater force range to increase the excitation. Test were run through the speed range in increments of 500 rpm. At each speed increment, the shaft was accelerated to the desired speed over a 5 s period, then maintained at that speed for 10 s. This ensured that the system weas in steady state either side of the impact excitation to perform the time-domain runout removal. Longer runs were more important at lower speeds due to the lower number of runout cycles reducing the capabilities of the subtraction error reduction.

The methodology for removing shaft runout was then applied to each steady state signal, and is shown for the 6 000 rpm case in Figure 11.

|  |
| --- |
| Chart, bar chart, histogram  Description automatically generated |
| Chart  Description automatically generated |
| Chart, histogram  Description automatically generated |

Figure 11 – Signal Runout Subtraction Results - 6000 rpm

Once the displacement signal for the impact decay has been isolated for each speed, the parameter identification workflow (see Figure 4) is used alongside the force signal from the modal hammer to calculate the impedance matrices.

# Additional Work for Chapter Completion

## Repeats of Hammer Test Results

* Initial investigations were run with multiple repeats of each steady state signal and impact. The displacement signal from the rig, even at low speeds, was showing non-cyclic and erratic behaviour of the shaft. Moreover, the runout signal peak to peak amplitude changed before and after impact. This issue has been resolved by re-machining bearing seats on the shaft as they were out of tolerance specified on the drawings.
* The new shaft is much more stable, and new tests are being conducted at present.
* The effects of lubricant up to speeds of 10 000 rpm are currently being run, alongside dry bearings with the same fixed preload to observe the difference – as in Chapter 3.

## FRF Results

* Processing of new results to obtain FRFs and identify bearing parameters.
* Some success showing an increase in bearing stiffness with speed, however repeat results are required to ensure confidence in these conclusions.
* To date, the impedance matrices for each speed have all been acquired.

## Refinement of written work

* Chapter requires general refinement to the written work and layout.

# References

[1] P. Dietl, “Damping and Stiffness Characteristics of Rolling Element Bearings-Theory and Experiment,” TU Vienna, 1997.

[2] P. Dietl, J. Wensing, and G. C. Van Nijen, “Rolling bearing damping for dynamic analysis of multi-body systems - Experimental and theoretical results,” *Proc. Inst. Mech. Eng. Part K J. Multi-body Dyn.*, vol. 214, no. 1, pp. 33–43, 2000, doi: 10.1243/1464419001544124.

[3] L. S. Andrés, J. Baker, and A. Delgado, “Rotordynamic force coefficients of a Hybrid brush seal: Measurements and predictions,” *J. Eng. Gas Turbines Power*, vol. 132, no. 4, pp. 1–7, 2010, doi: 10.1115/1.3159377.

[4] G. P. Fritzen, “Identification of mass, damping, and stiffness matrices of mechanical systems,” *J. Vib. Acoust. Trans. ASME*, vol. 108, no. 1, pp. 9–16, 1986, doi: 10.1115/1.3269310.

[5] M. J. Goodwin, “Experimental techniques for bearing impedance measurement,” *J. Manuf. Sci. Eng. Trans. ASME*, vol. 113, no. 3, pp. 335–342, 1991, doi: 10.1115/1.2899705.

[6] R. Nordmann and K. Schöllorn, “Identification of Stiffness and Damping Coefficients of Journal Bearings By Means of the Impact Method,” in *Proc. Vibrations in Rotating Machinery: 2nd International Conference, Inst. Mech. Eng., Cambridge, UK*, 1980, pp. 231–238.

[7] H. Massmann and R. Nordmann, “Some new results concerning the dynamic behavior of annular turbulent seals,” *NASA. Lewis Res. Cent. Instab. Rotating Mach.*, 1985.

[8] H. Questa, M. Mohammadpour, S. Theodossiades, C. P. Garner, S. R. Bewsher, and G. Offner, “Tribo-dynamic analysis of high-speed roller bearings for electrified vehicle powertrains,” *Tribol. Int.*, vol. 154, no. July 2020, p. 106675, Feb. 2021, doi: 10.1016/j.triboint.2020.106675.